Student Workbook

Unit 11 Review — Sequences, Series & Probability

Name: ______________________ Date: ______________ Class: ________

UNIT SUMMARY

Consolidate your mastery of sequences, series, counting principles, the Binomial Theorem, and probability with comprehensive mixed-topic practice.

Unit 11 Summary: Sequences, Series & Probability

$$a_n = a_1 + (n-1)d \mid a_n = a_1 \cdot r^{n-1} \mid P(E) = |E|/|S|$$

11.1	Sequences & Notation — explicit & recursive formulas; index notation a_n.
11.2	Arithmetic — $a_n=a_1+(n-1)d$. $S_n=n/2\cdot(a_1+a_n)$.
11.3	Geometric — $a_n=a_1\cdot r^{n-1}$. $S_n=a_1(1-r^n)/(1-r)$. $S_\infty=a_1/(1-r)$, $\lvert r\rvert<1$.
11.4	Series — sigma notation; partial sums; convergence.
11.5	Counting — $P(n,r)=n!/(n-r)!$. $C(n,r)=n!/[r!(n-r)!]$.
11.6	Binomial — $(a+b)^n=\Sigma C(n,k)a^{n-k}b^k$. k-th term: $C(n,k-1)a^{n-(k-1)}b^{k-1}$.
11.7	Probability — $P(E)=\lvert E\rvert/\lvert S\rvert$. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$. $P(A\mid B)=P(A\cap B)/P(B)$.

Worked Examples

EXAMPLE 1

Find a_{10} for the arithmetic sequence with $a_1 = 3$ and $d = 5$

- Use $a_n = a_1 + (n-1)d$
- $a_{10} = 3 + (10-1)(5) = 3 + 9\cdot5$
- $a_{10} = 3 + 45$

Answer: $a_{10} = 48$

EXAMPLE 2

Find S_∞ for the geometric series with $a_1 = 12$ and $r = 1/4$

- Check convergence: $|r| = 1/4 < 1$ ✓ — the series converges
- $S_\infty = a_1/(1-r) = 12/(1 - 1/4) = 12/(3/4)$
- $S_\infty = 12 \cdot (4/3)$

Answer: $S_\infty = 16$

EXAMPLE 3

A committee of 3 people is chosen from a group of 10. How many ways can this be done?

- Order does not matter → use combinations
- $C(10,3) = 10!/[3!\cdot 7!] = (10\cdot 9\cdot 8)/(3\cdot 2\cdot 1)$
- $C(10,3) = 720/6$

Answer: $C(10,3) = 120$ ways

EXAMPLE 4

Find the 4th term of $(x + 2)^5$ using the Binomial Theorem

- The k-th term is $C(n,k-1)\cdot a^{n-(k-1)}\cdot b^{k-1}$. For the 4th term, k=4, so use k−1=3
- $T_4 = C(5,3)\cdot x^{5-3}\cdot 2^3$
- $C(5,3) = 10$; $T_4 = 10\cdot x^2\cdot 8$

Answer: $T_4 = 80x^2$

EXAMPLE 5

$P(A) = 0.4$, $P(B) = 0.3$, and A and B are independent. Find $P(A\cap B)$ and $P(A\cup B)$

- Since A and B are independent: $P(A\cap B) = P(A)\cdot P(B)$
- $P(A\cap B) = 0.4 \cdot 0.3 = 0.12$
- $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.4 + 0.3 - 0.12$

Answer: $P(A\cap B) = 0.12$; $P(A\cup B) = 0.58$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Find a_8 for the geometric sequence with $a_1 = 2$ and $r = 3$.

Hint: Use $a_n = a_1\cdot r^{n-1}$ with n=8. Compute 3^7 first.

2 Find S_{10} for the arithmetic series with $a_1 = 1$ and $d = 4$.

Hint: First find $a_{10} = a_1 + 9d$, then use $S_n = n/2\cdot(a_1 + a_n)$.

3 Evaluate $P(8, 3)$.

Hint: $P(n,r) = n!/(n-r)! = 8!/5!$. Cancel the common factorial terms.

4 Expand $(x - 1)^3$ using the Binomial Theorem.

Hint: Write as $(x + (-1))^3$. Use C(3,0), C(3,1), C(3,2), C(3,3) for each term.

5 Two fair dice are rolled. What is the probability that the sum equals 7?

Hint: List all outcomes that sum to 7: (1,6),(2,5),(3,4),(4,3),(5,2),(6,1). Total outcomes = 36.

Key Vocabulary

Arithmetic Seq.	A sequence with a constant difference d between consecutive terms.
Geometric Seq.	A sequence with a constant ratio r between consecutive terms.
Partial Sum (S_n)	The sum of the first n terms of a sequence.
Infinite Series	The sum of infinitely many terms; converges when $\|r\| < 1$ for geometric series.
Permutation	P(n,r) — an ordered arrangement of r items chosen from n distinct items.
Combination	C(n,r) — an unordered selection of r items chosen from n distinct items.
Binomial Thm.	A formula for expanding $(a+b)^n$ as a sum using binomial coefficients C(n,k).
Sample Space	The set S of all possible outcomes of an experiment.
Conditional Prob.	$P(A\|B) = P(A\cap B)/P(B)$; the probability of A given that B has occurred.
Independent Events	Events where $P(A\cap B) = P(A)\cdot P(B)$; occurrence of one does not affect the other.

Quick Check

Circle the letter of the correct answer for each question.

1 **Which formula gives the sum of an infinite geometric series?** Multiple Choice

A $S_n = n/2 \cdot (a_1 + a_n)$
B $S_\infty = a_1/(1 - r)$, when $|r| < 1$
C $S_\infty = a_1 \cdot r/(1 - r)$
D $S_n = a_1(1 - r^n)/(1 - r)$

2 **How many ways can a president and vice-president be chosen from 8 candidates?** Multiple Choice

A C(8,2) = 28
B P(8,2) = 56
C 8 × 8 = 64
D C(8,2) = 56

Common Mistakes to Avoid

✗ Using $S_n = a_1(1-r^n)/(1-r)$ when $|r| \geq 1$ for an infinite sum.	✓ The infinite geometric series formula $S_\infty = a_1/(1-r)$ only applies when $|r| < 1$.
✗ Confusing P(n,r) and C(n,r) — using permutations when order does not matter.	✓ Use C(n,r) for committees, groups, or selections where order is irrelevant.
✗ Forgetting to subtract $P(A\cap B)$ in the addition rule.	✓ Always use $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ to avoid double-counting.
✗ Indexing the Binomial Theorem term incorrectly (off by one).	✓ The k-th term uses $C(n,k-1)$, so the 1st term uses C(n,0), the 2nd uses C(n,1), etc.

Math Tips

★ Write out the first few terms of any sequence to confirm your formula before computing large values.

★ For infinite geometric series, always check $|r| < 1$ before applying $S_\infty = a_1/(1-r)$.

★ Pascal's Triangle gives binomial coefficients quickly for small n: row n lists C(n,0) through C(n,n).

★ When counting, ask: does order matter? Yes → permutation. No → combination.

★ For probability problems with 'without replacement,' the denominator changes with each draw.